Predictive Momentum Management for the Space Station

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Space station control moment gyro momentum management is addressed by posing a deterministic optimization problem with a performance index that includes station external torque loading, gyro control torque demand, and excursions from desired reference attitudes. It is shown that a simple analytic desired attitude solution exists for all axes with pitch prescription decoupled, but roll and yaw coupled. Continuous gyro desaturation is shown to fit neatly into the scheme. Example results for pitch axis control of the NASA power tower space station are shown based on predictive attitude prescription. Control effector loading is shown to be reduced by this method when compared to more conventional momentum management techniques.

Introduction

LARGE space platforms such as the NASA-conceived power tower space station[1] are subject to a variety of disturbance torques, yet are expected to maintain attitude control over long intervals without resupply of consumables. Inclusion of nonpropulsive control effectors in the vehicle provides a means to reduce the demand for propellant for attitude maintenance although the nonpropulsive controller uses power that may come at the expense of increased drag and thus increased reboost propellant.

Momentum dumping devices include thrusters, magnetic torquers, and aerodynamic drag surfaces. Thrusters utilize propellant which we seek to avoid. Magnetic torquers in combination with reaction wheels have been used extensively for small satellite momentum management. The Space Telescope will use these effectors and avoid thrusters altogether. In the process of designing the Space Telescope system it has been recognized that a trade between momentum management software complexity and demands on system performance can be accommodated.[2] Schemes that recognize that orbit-averaged momentum storage is much smaller than cyclic variations have been devised to keep magnetic momentum dumping requirements small.[3] Models of the Earth's magnetic field and gravity-gradient effects are required for local vertical attitude control. Optimal regulator control schemes have been devised which mathematically minimize a combination of reaction-wheel momentum stored and magnetic torque applied that result in linear feedback gains and explicit design trade criteria.[4] It has also been recognized that a quadratic cost index should include the effector acceleration (or torque) term to avoid discontinuities in optimal momentum distribution in the reaction wheeels at the end of the solution interval,[5] although the case involving the additional performance measure was not solved. In the space station application there are complex effects of the large platform on the local magnetic field. Also, huge momentum unloading demands would be imposed on the low-authority effectors if similar momentum management schemes and effector design were used for the station as for the small satellites. These issues raise questions about the application of magnetic torquer technology to the space station.

Control moment gyros (CMGs) offer another flight-proven nonpropulsive attitude control technology with much greater momentum storage capacity and control torque authority than most reaction wheels. However, momentum capacity and control authority are still limited, making it necessary when using CMGs to either manage the commanded vehicle attitude to preclude effector saturation or to rely on devices that can dump unwanted stored momentum.

Space station momentum dumping can be avoided by effective manipulation of vehicle attitude within an acceptable orientation excursion zone. Environment torque loading then alters the CMG momentum state. Large transient disturbances, as from berthing vehicles or modules, or translating large masses along the station, may still require large capability momentum dumping effectors such as a reaction control system (RCS), but RCS use can be limited to quick recovery from large disturbances.

The NASA Skylab system used a combination of CMG and RCS effectors. However, unlike plans for the space station, most Skylab science objectives were met during daylight parts of each orbit using inertial attitudes for solar study and local vertical/local horizontal (LVLH) orientations for Earth observation. Night passes were generally available for attitude manipulation as required. Therefore, precise attitude hold was maintained in daylight with CMG secular momentum storage growth, and nightside attitude maneuvers were performed to remove stored momentum by gravity-gradient torque.[6,7]

The space station is expected to maintain LVLH orientation at all times. Furthermore, there has been a constraint imposed on acceptable attitude deviations for the space station between 1 and 5 deg.[8] This is usually interpreted as a constraint with respect to the current average torque balance attitude. Due to expected high power usage, the solar panel area will be very large, which makes aerodynamic torques important at operating altitude, especially with the large offset between the center of mass (CM) and center of pressure (CP) expected for the power tower.

Several design goals are apparent for space station CMG momentum management. CMG momentum storage requirements should be kept within the capability of a small number of the effectors to keep power and mass requirements low. Current baselines call for six dual-axis CMGs which must accomodate adequate control of the station with realistic failures in between servicing missions. Each can store about 5000 N-m-s momentum. Desaturation should be accommodated continuously within nominal control modes to preclude periodic interruption of ongoing station operations. RCS control should be the method of last resort for attitude control to prevent excess use of consumables.

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This paper presents analysis and results for a particular momentum management concept applied to the power tower space station. System dynamics are expressed and linearized in a manner appropriate to a large LVLH-oriented platform, and it is shown that with the simplified equations of motion the pitch dynamics are decoupled from other axes, but roll and yaw remain coupled. A deterministic optimal control problem is then posed with a quadratic performance index that includes as variables space station torque loading, CMG torque demand, and attitude deviations from a desired reference. These quantities are weighted with gain coefficients. Both the uncoupled pitch and coupled roll-yaw optimization problems are shown to lead to differential equations which express predicted attitude histories. A conceptual control system design including predictive momentum management is then outlined. The attitude predictions can be made once per orbit, with a desaturation term included in the solution to prevent unwanted secular momentum storage in the future and to remove any accrued on previous orbits due to unmodeled effects. Since the pitch axis is the dominant control problem for the power tower, it is selected to demonstrate the application of the predictive technique. Finally, it is shown that the technique leads to more effective CMG use than conventional momentum management approaches.

Problem Characterization

Before developing a momentum management strategy, it is necessary to characterize the environment model. The important environment torques are due to gravity gradients and aerodynamic drag.

The aerodynamic drag torque exhibits variations due to both atmospheric density fluctuations and space station configuration changes. Atmospheric density variations take place at several frequencies.[9] There is a factor of 100 variation over the 11-year solar activity cycle; a factor of 2-4 variation over the annual, semiannual, and 27-day solar cycles; up to a factor of 3 variation on a few days' scale due

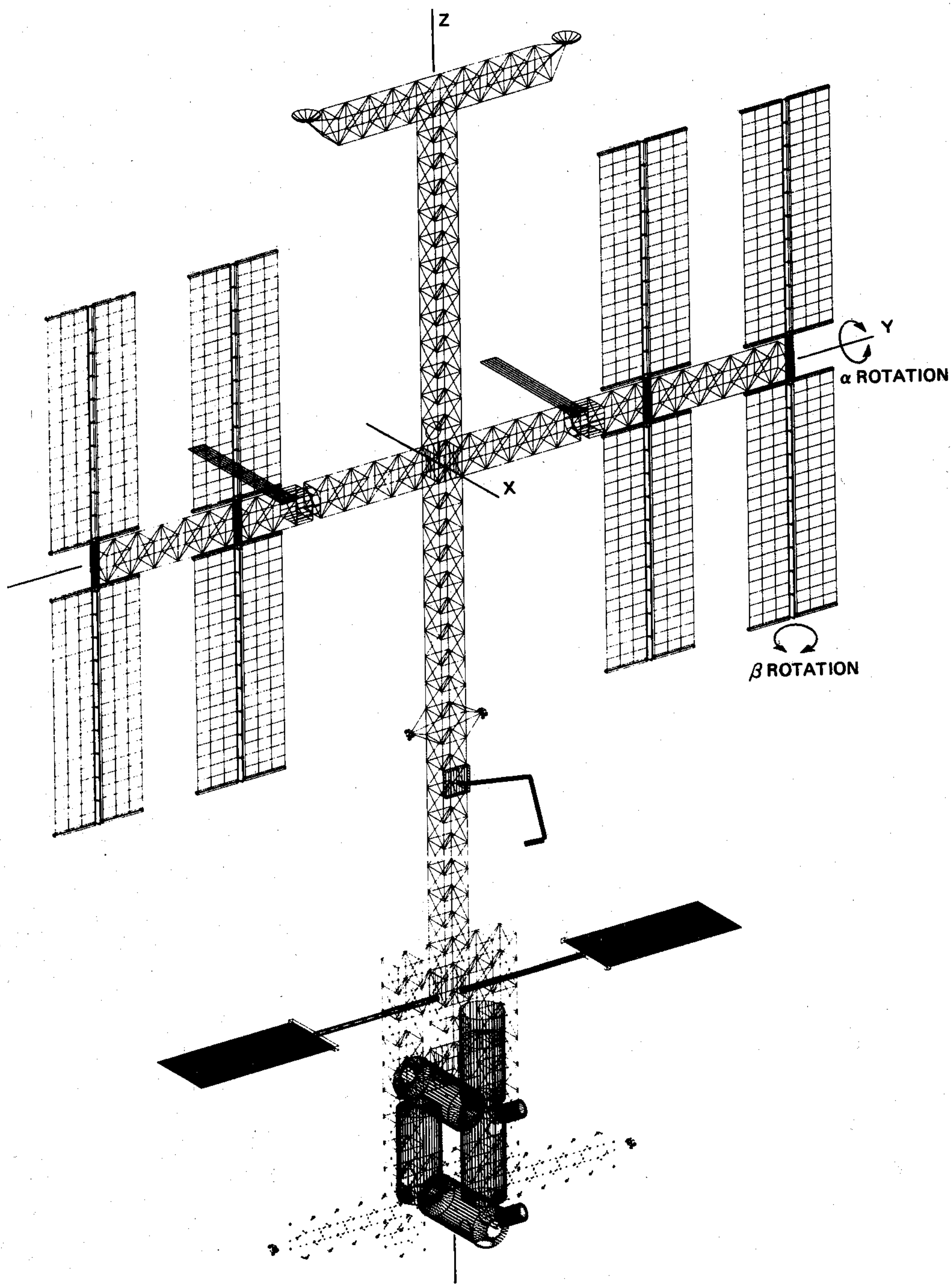

Fig. 1 Power tower space station.

to geomagnetic storms; and, finally, there is a once-per-orbit variation due to the solar-heating-induced diurnal bulge. Density fluctuations at frequencies higher than orbital are possible, but have not been adequately characterized by observation. For purposes of this study the density fluctuations are modeled as a low-order Fourier series in orbital frequencies with slowly varying coefficients, based on Ref. 10.

The 1971 Jacchia atmosphere model prediction for January 1, 1982, was chosen as the basis for computer runs, which characterize the power tower momentum management problem. (The date is close to the 11-yr solar cycle peak.) The power tower, depicted in Fig. 1, was assumed to have mass properties given in Table 1 along with the assumed solar panel surface area and orbit altitude. The starting point for each simulated orbit was the atmosphere minimum density point. Drag effects of elements other than solar panels were ignored. Pitch axis results are plotted in an LVLH frame. The solar panels on the power tower have two rotational degrees of freedom to track the sun, as indicated in Fig. 1.

Figure 2 shows, for comparison purposes, baseline simulation results. Momentum storage (in pitch) is shown for three pitch attitude sequences. Figures 2a and 2b show the effects of starting in an Earth-pointing LVLH attitude, and flying in free drift with rotating (sun-tracking) solar panels. Over one orbit, the peak attitude errors are well in excess of what is likely to be acceptable, and they span a 10-deg range.

Figure 2d shows the stored momentum history over one orbit when the power tower is held in an Earth-pointing LVLH attitude (Fig. 2c). Since one CMG of the type expected for the space station stores a maximum of about 5000 N-m-s momentum, the results suggest that complete desaturation of all six CMGs (if fully operable) could be required twice per orbit just for in-orbit-plane control, which is unacceptably demanding.

Figure 2f shows the CMG momentum storage history when flying a constant attitude (Fig. 2e) offset from LVLH Earth pointing to achieve an average torque balance over the orbit. In this case the offset is about 1.84 deg. The advantage of this attitude sequence is acceptable net CMG loading. However, it presumes nearly perfect knowledge of atmospheric state to prevent secular momentum storage growth. Since perfect environment knowledge is not likely in real application, a means to desaturate CMGs due to unmodeled effects must be provided. It may also be desirable to limit CMG loading peaks further even though the orbit net loading is zero.

As goals, attitude variations less than ±2 deg about the current average torque balance attitude, with peak momentum storage of less than one CMG's limit is desired per axis. This would accommodate control error and actual momentum storage uncertainties without changing current design requirements. Another control strategy is required to meet these objectives. A predictive momentum management concept can be devised to address these design goals.

Table 1 Parameters used for simulation (for "bare," initial operational configuration power tower)

Parameter	Value
Roll inertia I_x, $kg-m^2$	9.872×10^7
Pitch inertia I_y, $kg-m^2$	9.471×10^7
Yaw inertia I_z, $kg-m^2$	7.549×10^6
Solar panel surface area, m^2	1983
Solar panel drag coefficient, C_D	2.7
CM to CP distance (along z), m	56
Orbit altitude, km	480

Linearizing the System Dynamics

Suppose that the space station body axes in an LVLH frame are coincident with the principal axes of inertia, I_x, I_y, and I_z. (For the power tower this is a very good assumption for pitch, and a fair one for roll and yaw.) From elementary mechanics one has

$$\Sigma M_x = I_x\phi^{(2)} - (I_y - I_z)\theta^{(1)}\psi^{(1)} \tag{1a}$$

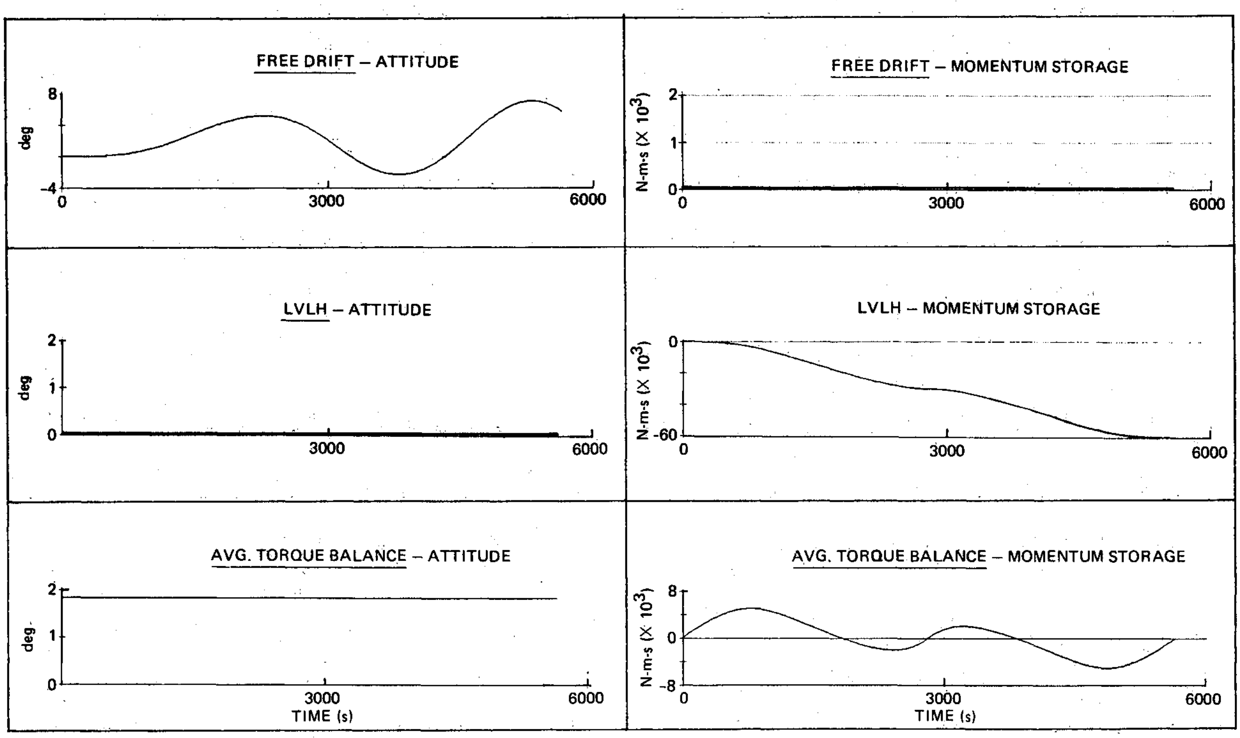

Fig. 2 Space station pitch momentum management performance results for comparative nonpredictive strategies (1971 Jacchia atmosphere model for January 1, 1982).

$$\Sigma M_y = I_y\theta^{(2)} - (I_z - I_x)\psi^{(1)}\phi^{(1)} \tag{1b}$$

$$\Sigma M_z = I_z\psi^{(2)} - (I_x - I_y)\phi^{(1)}\theta^{(1)} \tag{1c}$$

where M_i is the moment around axis i, and ϕ, θ, ψ are the roll, pitch, and yaw rotations, respectively; parenthetical superscripts represent the time derivative order.

For nominal space station operations, control rates are never expected to exceed 0.002 to 0.003 deg/s. The inertial pitch rate is about 0.067 deg/s, however, to maintain an LVLH attitude on average. Denote the orbit rate as ω_{orb}. The products of control rates are very small compared to the products of orbit rate with control rates. To first order this yields

$$\Sigma M_x = I_x\phi^{(2)} - (I_y - I_z)\omega_{\text{orb}}\psi^{(1)} \tag{2a}$$

$$\Sigma M_y = I_y\theta^{(2)} \tag{2b}$$

$$\Sigma M_z = I_z\psi^{(2)} - (I_x - I_y)\omega_{\text{orb}}\phi^{(1)} \tag{2c}$$

Inspection of Eqs. (2) shows that the equations are linear, pitch is uncoupled, but roll and yaw remain coupled.

A Basis for Predictive Momentum Management

The physical quantities most important for space station momentum management are the momentum buildup, the CMG control torque demand, and the attitude excursion from a desired reference. Combining these terms in a quadratic performance index leads to a basis for choosing a desired attitude history that constrains CMG momentum loading. In the process, a predicted momentum storage history can be derived which provides a guideline for continuous gyro desaturation. The process is started by defining two performance indices—one for the pitch problem and one for the roll-yaw problem.

$$J_\theta = \int_0^{t_f} [\tau_{\text{ext}\theta}^2 + K_{\theta 1}\tau_{\text{CMG}\theta}^2 + K_{\theta 2}(\theta - \theta_{\text{ref}}(t))^2]\,\mathrm{d}t \tag{3a}$$

$$J_\phi = \int_0^{t_f} [(\tau_{\text{ext}\phi}^2 + \tau_{\text{ext}\psi}^2) + K_{\phi 1}(\tau_{\text{CMG}\phi}^2 + \tau_{\text{CMG}\psi}^2) + K_{\phi 2}(\phi - \phi_{\text{ref}}(t))^2 + K_{\phi 3}(\psi - \psi_{\text{ref}}(t))^2]\,\mathrm{d}t \tag{3b}$$

where

J_θ, J_ϕ = pitch and roll-yaw performance indices

$\tau_{\text{ext}\alpha}$ = expected external torque loading on the space station in rotation axis α

$\tau_{\text{CMG}\alpha}$ = CMG expected torque demand in rotation axis α

$\theta_{\text{ref}}(t), \phi_{\text{ref}}(t), \psi_{\text{ref}}(t)$ = prescribed attitude offset histories from Earth-pointing LVLH (expected to be small angles)

$K_{\theta i}, K_{\phi i}$ = design constants for the pitch and roll-yaw problems with dimensions as necessary to make performance index terms similar

t_f = time span over which the evaluation will be done (one or more orbits)

It is desirable to define variables that put Eqs. (3) and their associated minimization in canonical form.

$$x_{\theta 1} = \theta, \quad x_{\theta 2} = \theta^{(1)}, \quad u_\theta = \tau_{\text{CMG}\theta} \tag{4a}$$

$$x_{\phi 1} = \phi, \quad x_{\phi 2} = \psi, \quad x_{\phi 3} = \phi^{(1)}, \quad x_{\phi 4} = \psi^{(1)}$$

$$u_\phi = \tau_{\text{CMG}\phi}, \quad u_\psi = \tau_{\text{CMG}\psi} \tag{4b}$$

The external torques are a sum of gravity-gradient and aerodynamic effects. To first order, for small attitude offsets, the gravity gradients are linear with offset attitude in pitch and roll, and aerodynamic torques are time-dependent, but attitude-independent functions of orbit position and solar panel orientation.

$$\tau_{\text{ext}\theta} = C_\theta x_{\theta 1} + a_\theta(t)$$

$$\tau_{\text{ext}\phi} = C_\phi x_{\phi 1} + a_\phi(t)$$

$$\tau_{\text{ext}\psi} = a_\psi(t) \tag{5}$$

where C_θ and C_ϕ are the gravity-gradient torque coefficients for pitch and roll, and $a_\alpha(t)$ the aerodynamic torque expected for rotation axis α. From our model we have

$$\Sigma M_i = \tau_{\text{ext}\alpha} + \tau_{\text{CMG}\alpha} \tag{6}$$

Hamiltonians can be constructed from the performance index integrands and the state time derivatives by introducing costate vector elements $\lambda_{\theta i}$ and $\lambda_{\phi i}$. Equations (2-6) lead to the following:

$$H_\theta = (C_\theta x_{\theta 1} + a_\theta(t))^2 + K_{\theta 1}u_\theta^2 + K_{\theta 2}(x_{\theta 1} - \theta_{\text{ref}}(t))^2 + \lambda_{\theta 1}x_{\theta 2} + \lambda_{\theta 2}(C_\theta x_{\theta 1} + a_\theta(t) + u_\theta)/I_y \tag{7a}$$

$$H_\phi = ([C_\phi x_{\phi 1} + a_\phi(t)]^2 + a_\psi^2(t)) + K_{\phi 1}(u_\phi^2 + u_\psi^2) + K_{\phi 2}(x_{\phi 1} - \phi_{\text{ref}}(t))^2 + K_{\phi 3}(x_{\phi 2} - \psi_{\text{ref}}(t))^2 + \lambda_{\phi 1}x_{\phi 3} + \lambda_{\phi 2}x_{\phi 4} + \lambda_{\phi 3}(C_\phi x_{\phi 1} + a_\phi(t) + u_\phi + [I_y - I_z]\omega_{\text{orb}}x_{\phi 4})/I_x + \lambda_{\phi 4}(a_\psi(t) + u_\psi + [I_x - I_y]\omega_{\text{orb}}x_{\phi 3})/I_z \tag{7b}$$

Minimizing the Hamiltonians in Eqs. (7) with respect to the controls leads to the following expected CMG control history:

$$u_\theta = \frac{-\lambda_{\theta 2}}{2K_{\theta 1}I_y}$$

$$u_\phi = \frac{-\lambda_{\phi 3}}{2K_{\phi 1}I_x}$$

$$u_\psi = \frac{-\lambda_{\phi 4}}{2K_{\phi 1}I_z} \tag{8}$$

Note that the CMG torque demand is inversely proportional to the performance index weightings on the τ_{CMG} terms.

Use of Eqs. (2), (4-6), and (8) provide the basis for deriving three coupled differential equations.

$$x_{\theta 1}^{(2)} - \left(\frac{C_\theta}{I_y}\right)x_{\theta 1} = -\left(\frac{1}{2K_{\theta 1}I_y^2}\right)\lambda_{\theta 2} + \frac{a_\theta(t)}{I_y} \tag{9a}$$

$$x_{\phi 1}^{(2)} - \left(\frac{C_\phi}{I_x}\right)x_{\phi 1} = \left(\frac{I_y - I_z}{I_x}\right)\omega_{\text{orb}}x_{\phi 2}^{(1)} - \left(\frac{1}{2K_{\phi 1}I_x^2}\right)\lambda_{\phi 3} + \frac{a_\phi(t)}{I_x} \tag{9b}$$

$$x_{\phi 2}^{(2)} = \left(\frac{I_x - I_y}{I_z}\right)\omega_{\text{orb}}x_{\phi 1}^{(1)} - \left(\frac{1}{2K_{\phi 1}I_z^2}\right)\lambda_{\phi 4} + \frac{a_\psi(t)}{I_z} \tag{9c}$$

Optimal control theory provides the result that $\dot{\lambda}^{(1)} = -\partial H/\partial \bar{x}$, which leads to the following differential equations for $\bar{\lambda}$:

$$-\lambda_{\theta 1}^{(1)} = 2C_\theta (C_\theta x_{\theta 1} + a_\theta (t)) + 2K_{\theta 2}(x_{\theta 1} - \theta_{\text{ref}}(t)) + \lambda_{\theta 2} C_\theta / I_y \tag{10a}$$

$$-\lambda_{\theta 2}^{(1)} = \lambda_{\theta 1} \tag{10b}$$

$$-\lambda_{\phi 1}^{(1)} = 2(C_\phi^2 + K_{\phi 2})x_{\phi 1} + (C_\phi / I_x)\lambda_{\phi 3} + 2(C_\phi a_\phi (t) - K_{\phi 2}\phi_{\text{ref}}(t)) \tag{10c}$$

$$-\lambda_{\phi 2}^{(1)} = 2K_{\phi 3}x_{\phi 2} - 2K_{\phi 3}\psi_{\text{ref}}(t) \tag{10d}$$

$$-\lambda_{\phi 3}^{(1)} = \lambda_{\phi 1} + \left(\left[\frac{I_x - I_y}{I_z}\right]\omega_{\text{orb}}\right)\lambda_{\phi 4} \tag{10e}$$

$$-\lambda_{\phi 4}^{(1)} = \lambda_{\phi 2} + \left(\left[\frac{I_y - I_z}{I_x}\right]\omega_{\text{orb}}\right)\lambda_{\phi 3} \tag{10f}$$

Equation (10b) can be differentiated and used to eliminate $\lambda_{\theta 1}$ from Eq. (10a). Equations (10e) and (10f) can be differentiated and used to eliminate $\lambda_{\phi 1}$ and $\lambda_{\phi 2}$ from Eqs. (10c) and (10d). Expressions in terms of state variables can be derived from Eqs. (9) for $\lambda_{\theta 2}$, $\lambda_{\phi 3}$, and $\lambda_{\phi 4}$ to be used to eliminate them from Eqs. (10) and derive a differential equation set in terms of the state variables alone. The following equations result:

$$\theta^{(4)} - S_{\theta 1}\theta^{(2)} + S_{\theta 2}\theta = A_\theta (t) \tag{11a}$$

$$\phi^{(4)} - S_{\phi 1}\phi^{(2)} + S_{\phi 2}\phi = S_{\phi 3}\psi^{(3)} - S_{\phi 4}\psi^{(1)} + A_\phi (t) \tag{11b}$$

$$\psi^{(4)} - S_{\phi 5}\psi^{(2)} + S_{\phi 6}\psi = S_{\phi 7}\phi^{(3)} + S_{\phi 8}\phi^{(1)} + A_\psi (t) \tag{11c}$$

where the constants $S_{\alpha i}$ are defined in Table 2 and the expressions for the drivers $A_\alpha(t)$ are in Table 3. If r_θ and r_ϕ represent the solution roots for the pitch and roll-yaw dynamics expressed in Eqs. (11), it can be shown that

$$r_\theta^4 - S_{\theta 1} r_\theta^2 + S_{\theta 2} r_\theta = 0 \tag{12a}$$

$$r_\phi^8 - (S_{\phi 1} + S_{\phi 5} + S_{\phi 3}S_{\phi 7}) r_\phi^6 + (S_{\phi 1}S_{\phi 5}) r_\phi^4 + (S_{\phi 4}S_{\phi 8}) r_\phi^2 + S_{\phi 2}S_{\phi 6} = 0 \tag{12b}$$

Note that the pitch equation is quadratic in terms of r_θ^2 and the roll-yaw equation is quartic in terms of r_ϕ^2. In both cases, analytic expressions for root evaluation exist.

Based on Ref. 10, it is reasonable to assume characterization of the aerotorque driving terms in Eqs. (11) by a small number of Fourier series terms that describe the diurnal bulge. The coefficients, based upon latest density estimates, are assumed quasistatic. The nonhomogeneous terms in the attitude state equations can then be obtained analytically. For pitch motion in Eq. (11a), since only even derivatives are present, a sine or cosine driving term leads to a similar sine or cosine solution term. For coupled roll-yaw motion, a sine or cosine driving term in one axis yields a term of common

Table 2 Differential equation coefficients

Constant	Expression for evaluation
$S_{\theta 1}$	$\frac{2C_\theta}{I_y}$
$S_{\theta 2}$	$\left(\frac{C_\theta^2}{K_{\theta 1} I_y^2}\right)\left(1 + K_{\theta 1} + \frac{K_{\theta 2}}{C_\theta^2}\right)$
$S_{\phi 1}$	$\frac{2C_\phi}{I_x} + \left(\frac{I_x - I_y}{I_x}\right)^2 \omega_{\text{orb}}^2$
$S_{\phi 2}$	$\left(\frac{C_\phi^2}{K_{\phi 1} I_x^2}\right)\left(1 + K_{\phi 1} + \frac{K_{\phi 2}}{C_\phi^2}\right)$
$S_{\phi 3}$	$\omega_{\text{orb}}\left(\left[\frac{I_y - I_z}{I_x}\right] - \left[\frac{I_x - I_y}{I_z}\right]\left[\frac{I_z}{I_x}\right]^2\right)$
$S_{\phi 4}$	$C_\phi \omega_{\text{orb}}\left(\frac{I_y - I_z}{I_x^2}\right)$
$S_{\phi 5}$	$\left(\frac{I_y - I_z}{I_z}\right)^2 \omega_{\text{orb}}^2$
$S_{\phi 6}$	$\left(\frac{1 - K_{\phi 3}}{K_{\phi 1}}\right)\left(\frac{1}{I_z^2}\right)$
$S_{\phi 7}$	$\omega_{\text{orb}}\left(\left[\frac{I_x - I_y}{I_z}\right] - \left[\frac{I_y - I_z}{I_x}\right]\left[\frac{I_x}{I_z}\right]^2\right)$
$S_{\phi 8}$	$C_\phi \omega_{\text{orb}}\left(\frac{I_y - I_z}{I_z^2}\right)$

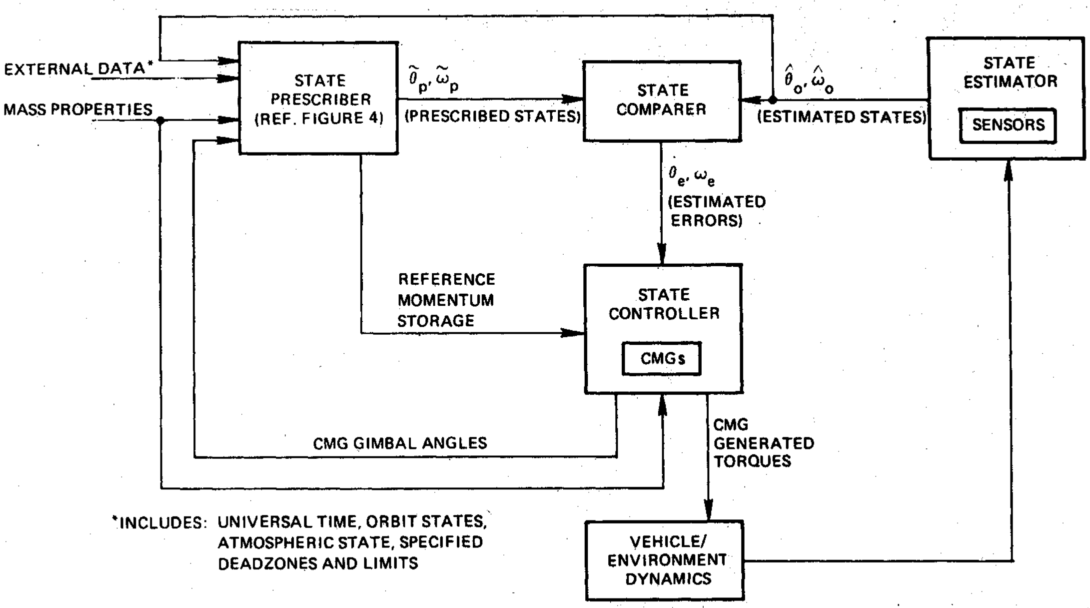

Fig. 3 Conceptual momentum management system.

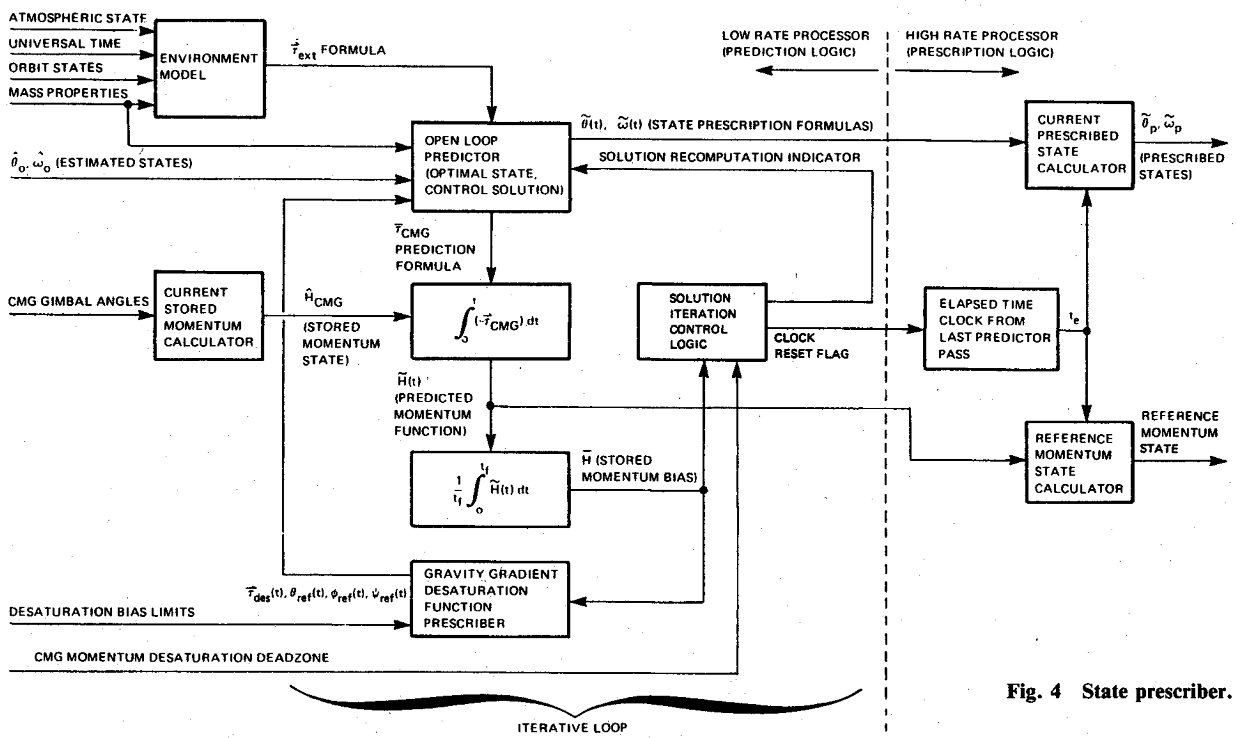

Fig. 4 State prescriber.

phase in the solution for that axis and a term with 90-deg phase shift in the other axis. [This is due to only even ϕ derivatives and odd ψ derivatives in Eq. (11b) with a complementary pattern in Eq. (11c).]

To complete the solution process for the optimal attitude sequence differential equations, it is necessary to determine 12 boundary conditions that provide a basis for evaluating coefficients for the solution terms resulting from the homogeneous equation roots for Eqs. (12). Six of the boundary conditions are the estimated attitude and angular rate at the beginning of the optimization solution interval (where $t=0$ is assumed). The other six are derived from the optimal control condition that the costates evaluated at final time have zero value. This result can be combined with Eqs. (9) and (10) to eliminate costate dependence in the remaining boundary conditions. They are

$$\theta^{(2)}(t_f)-\frac{C_\theta}{I_y}\theta(t_f)=a_\theta(t_f)/I_y \tag{13a}$$

$$\theta^{(3)}(t_f)-\frac{C_\theta}{I_y}\theta^{(1)}(t_f)=a_\theta^{(1)}(t_f)/I_y \tag{13b}$$

$$\phi^{(2)}(t_f)-\frac{C_\phi}{I_x}\phi(t_f)=\left(\frac{I_y-I_z}{I_x}\right)\omega_{\rm orb}\psi^{(1)}(t_f)+\frac{a_\phi(t_f)}{I_x} \tag{13c}$$

$$\phi^{(3)}(t_f)-\frac{C_\phi}{I_x}\phi^{(1)}(t_f)=\left(\frac{I_y-I_z}{I_x}\right)\omega_{\rm orb}\psi^{(2)}(t_f)+\frac{a_\phi^{(1)}(t_f)}{I_x} \tag{13d}$$

$$\psi^{(2)}(t_f)=\left(\frac{I_x-I_y}{I_z}\right)\omega_{\rm orb}\phi^{(1)}(t_f)+\frac{a_\psi(t_f)}{I_z} \tag{13e}$$

$$\psi^{(3)}(t_f)=\left(\frac{I_x-I_y}{I_z}\right)\omega_{\rm orb}\phi^{(2)}(t_f)+\frac{a_\psi^{(1)}(t_f)}{I_z} \tag{13f}$$

Having determined a basis for analytically evaluating a predicted state without explicitly addressing the problem of constraining CMG momentum storage, it is necessary to establish a basis for desaturation. Note that Eqs. (8) and (9) can be manipulated to eliminate the costates and solve for a predicted control history in terms of predicted attitude states and expected aerodynamic driving functions.

Table 3 Differential equation driving terms

Driver	Formula
$A_\theta(t)$	$\left(\frac{1}{I_y}\right)a_\theta^{(2)}(t)-\left(\frac{C_\theta}{K_{\theta 1}I_y^2}\right)(1+K_{\theta 1})a_\theta(t)+\frac{K_{\theta 2}}{K_{\theta 1}I_y^2}\theta_{\rm ref}(t)$
$A_\phi(t)$	$\left(\frac{1}{I_x}\right)a_\phi^{(2)}(t)-\left(\frac{C_\phi}{K_{\phi 1}I_x^2}\right)(1+K_{\phi 1})a_\phi(t)$ $+\omega_{\rm orb}\left(\frac{I_x-I_y}{I_x^2}\right)a_\psi^{(1)}(t)+\left(\frac{K_{\phi 2}}{K_{\phi 1}I_x^2}\right)\phi_{\rm ref}(t)$
$A_\psi(t)$	$\left(\frac{1}{I_z}\right)a_\psi^{(2)}(t)+\omega_{\rm orb}\left(\frac{I_y-I_z}{I_z^2}\right)a_\phi^{(1)}(t)$ $+\left(\frac{K_{\phi 3}}{K_{\phi 1}I_z^2}\right)\psi_{\rm ref}(t)$

$$u_\theta=I_y\theta^{(2)}-C_\theta\theta-a_\theta(t) \tag{14a}$$

$$u_\phi=I_x\phi^{(2)}-C_\phi\phi-(I_y-I_z)\omega_{\rm orb}\psi^{(1)}-a_\phi(t) \tag{14b}$$

$$u_\psi=I_z\psi^{(2)}-(I_x-I_y)\omega_{\rm orb}\phi^{(1)}-a_\psi(t) \tag{14c}$$

Since the solutions for the attitude states are composed of linear, exponential and linear, trigonometric terms, simple analytic expressions for the expected CMG torque demand, of similar form, result from Eqs. (14). A predicted momentum state of the CMGs can be derived analytically by an integration of Eqs. (14), resulting in another linear, exponential and linear, trigonometric-term equation.

$$\tilde{H}_\alpha=\int_0^t(-u_\alpha)\mathrm{d}t \tag{15}$$

where $\tilde{H}_\alpha$ is the predicted momentum CMG state for rotation axis α, and u_α the expected CMG control torque history for axis α. Complete evaluation of Eq. (15) requires a reading of the CMG momentum state at $t=0$ to provide an initial condition which includes unmodeled effects from the past. An additional integration can then provide an average predicted momentum state over the optimization interval which can be used as a desaturation guideline.

$$\bar{H}_\alpha = \frac{1}{t_f}\int_0^{t_f} \bar{H}_\alpha(t)\mathrm{d}t \tag{16}$$

where $\bar{H}_\alpha$ is the average expected momentum storage for axis α over the prediction interval. The evaluation of $\bar{H}$ in Eq. (16) provides a measurement of momentum to be removed. The shape of the solution of Eq. (15) provides a guideline for establishing a desired desaturation history, which helps keep peak momentum storage low. The desired desaturation function can then be used to generate a revised predicted attitude state history by reconsidering Eqs. (3). Let reference attitude histories in Eqs. (3) [e.g., $\theta_{\mathrm{ref}}(t)$] reflect the desaturation requirement and redefine the external torques:

$$\tau'_{\mathrm{ext}\alpha} = \tau_{\mathrm{ext}\alpha} + \tau_{\mathrm{des}\alpha} \tag{17}$$

where $\tau_{\mathrm{ext}_\alpha}$ is the modified external torque in axis α, and $\tau_{\mathrm{des}_\alpha}$ the desaturation torque history sought in axis α.

Resolution of the optimization problem results in the following:

$$u'_\alpha = u_\alpha + \tau_{\mathrm{des}\alpha} \tag{18}$$

where u'_α is the predicted control demand which includes continuous desaturation for axis α.

The modified predicted attitude equations have the same form as before, except that the aerodynamic driving terms are replaced by drivers which include desaturation.

$$a'_\alpha(t) = a_\alpha(t) + \tau_{\mathrm{des}_\alpha}(t) \tag{19}$$

where $a'_\alpha(t)$ is the modified driving term for axis α to be used in Eqs. (5) to include desaturation.

The benefit of the analytic predicted state equations becomes apparent as efforts are made to construct a conceptual control system including a state prescription. Figure 3 outlines a closed-loop vehicle control system design intended to include an attitude state prescription law for momentum management. Figure 4 details the prescription law which makes predictions from the analytic formulas derived in this paper. The prescription neatly partitions into a low- and high-rate processor. The low-rate processor would compute the predicted attitude formulas about once an orbit. The computation would be iterative to allow shaping a desaturation function which adequately removes expected and previously unmodeled secular momentum storage. Usually two, and rarely more than three or four iterations, are required. The high-rate processor assigns values to the attitude formulas to provide a basis for commanding the CMG steering law. In addition, a reference CMG momentum state vector is generated to provide a basis for determining if more potent effectors, such as RCS, need to be commanded due to

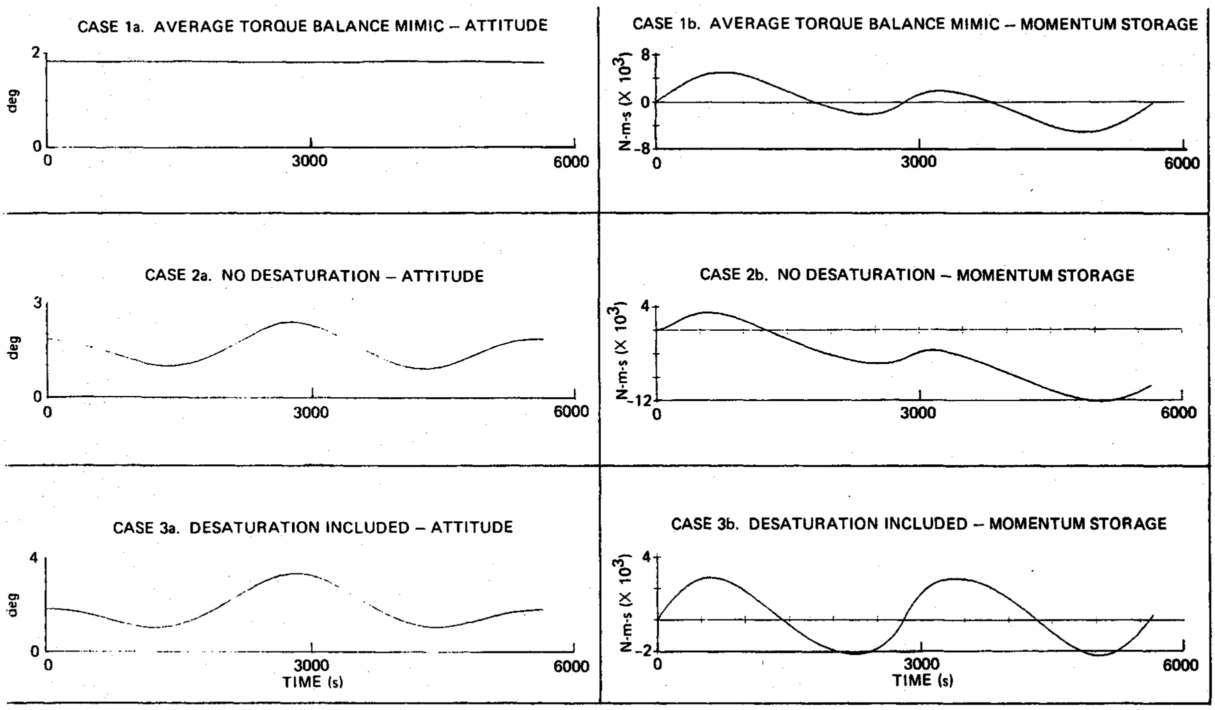

Fig. 5 Space station pitch momentum management results for various predictive cases (1971 Jacchia atmosphere model for January 1, 1982).

Table 4 Pitch study case parameters

Case No.	$K_{\theta 1}$	$K_{\theta 2}$, N^2-m^2	θ_{ref}, deg[a] Constant component	θ_{ref}, deg[a] Desaturation time function component ($t=0$ at start of solution interval)
1	1	1×10^7	1.84	None
2	19	1×10^6	1.84	None
3	19	1×10^6	1.84	$0.29\,(1-2\cos\omega_{\mathrm{orb}}t+\cos 2\,\omega_{\mathrm{orb}}t)$

[a] $\theta_{\mathrm{ref}}(t)$ is the sum of the constant and desaturation components.

large and/or unexpected disturbances which drive the actual CMG momentum state unacceptably far from the reference.

Some Pitch Axis Study Results

The power tower is expected to maintain a near-Earth-pointing LVLH configuration by rotating around its pitch axis at orbit rate on average. The major aerodynamic torque loading on the station will be due to solar panel drag along the velocity vector. Because of a high degree of horizontal symmetry, and because the center of pressure is far above the center of mass, the dominant CMG momentum management problem is in the pitch axis, which is dynamically decoupled from roll and yaw in our model. An illustration of the behavior and benefits of predictive momentum management can be provided by implementing a single-axis control loop for pitch.

Assignment of gains $K_{\theta 1}$ and $K_{\theta 2}$ introduced in the performance index [Eq. (3a)] is a necessary part of the analysis. Some experimentation was necessary to find values that resulted in sensitivity for a trade between peak CMG momentum storage and variations in space station pitch attitude over an orbit. For CMG steering, a responsive proportional-integral-differential algorithm was implemented to generate torques necessary to track the prescribed rotation states. For demonstration purposes no atmospheric modeling uncertainty was included. Table 4 lists the study parameters, and Fig. 5 plots the results.

Case 1 was chosen to show that the proper choice of gains causes the predictive momentum management system to mimic the average torque balance case shown earlier. Case 2 is the result of a run without desaturation included. Even in this case the attitude oscillations are held to about ± 0.7 deg with a one-orbit secular momentum storage growth about one-fifth of the result for precise Earth-pointing LVLH tracking. Case 3 shows the result for case 2 repeated with desaturation included simultaneously. The result is about half the peak momentum storage of the average torque balance case, peak attitude variations of about ± 1.17 deg, and no significant secular momentum storage component.

The results of the pitch study suggest that substantial reduction in peak CMG loading can be achieved with predictive momentum management by maintaining controlled attitude oscillations well within the NASA design recommendation, provided that the continuous desaturation scheme made possible by the analytical expected momentum solution is implemented. These conclusions are expected to be valid under all flight conditions since the test cases were run under near-maximum atmospheric density conditions (January 1, 1982, atmosphere from 1971 Jacchia model).

Some Further Study Requirements

In the work presented in this paper it has been assumed that the principal inertia terms are virtually the same as body-axis inertias (small cross products), and the inertias were presumed to be constant over the prediction interval. Some discrepancies with these assumptions in roll and yaw are likely due to imperfect solar panel rotation to track the sun, leading to some coupling of the moving solar panel mass into the controlled tower mass properties. The magnitude of the prediction errors that result needs to be evaluated. If the space station design changes substantially from the power tower configuration, then the assumptions made must be carefully reassessed.

The aerodynamic torque model was assumed to be well known over the prediction time interval for the analysis done here. A provision was included, however, to produce a reference momentum state that can be used to track modeling errors by maintaining knowledge of the difference between predicted momentum storage and actual CMG state. The basis for using this information to correct the atmospheric model in real time needs further investigation.

Conclusions

A basis for doing space station control moment gyro momentum management predictively has been shown. The dynamics have been linearized in a manner that decouples pitch from other axes but retains coupling effects between roll and yaw. Two performance indices have been defined which include control effector loading and space station attitude variation penalties. The solution of the deterministic optimal control problem of minimizing the performance indices leads to differential equations of desired station attitude which have analytic solutions. A simple conceptual control system using the prescribed attitudes as a predictive momentum management scheme was devised, and includes a continuous desaturation mechanization. A pitch axis control loop implementing the predictive logic was constructed and tested with results that show a substantial reduction in gyro loading compared to more conventional nonpredictive momentum management concepts while maintaining acceptable small station attitude oscillations.

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